Counting and Statistics Problem Set (Week of 10/25/11)

These problems vary in difficulty, so try all of them… Also, remember that you will be getting points for correct answers! I’d suggest working with others if you get a chance ☺.

1. Kennesaw. How many different arrangements can be made using all the letters of the name KENNESAW?
2. Lassiter. Aida, Omolio, Akshaya, Kim, Niti, and THE SUSHINATOR are all going to eat lunch at a round table for six. Unfortunately, Aida, Akshaya, and Niti hate each other, so no two of them can sit together. (If you know them, then the previous statement is unbelievably ironic.) Furthermore, Aida insists on sitting in her one, immovable special chair. How many different seating arrangements are possible? (Everyone cares about right and left; two arrangements are considered different if they are reflections of each other.)
3. Lassiter. Harrison has a 6x8 array of boxes (6 rows, 8 columns). In each row, he picks two boxes at random and puts a vase in each. Then he selects a column at random. What is the expected number of boxes in this column that contain vases?
4. Lassiter. Zac got 8 dolls for his birthday, 4 Barbie dolls and 4 Bratz dolls. Although she got 2 identical Holiday Barbie dolls, the other 2 Barbie dolls are unique. And although she got 2 identical Angel Bratz dolls, the other 2 Bratz dolls are unique. In how many distinguishable ways can she line up these 8 dolls on a shelf if she wants to alternate the Barbie and Bratz dolls?
5. What is the arithmetic mean of 456, 234, and 789?
6. If x + y = 10 and xy = 24, what is x2 + y2?
7. What is 10545 in base 25? In base 20?
8. What are the sum and product of the roots of 4x2 – 6x +7?
9. N. Fulton. How many 4 digit numbers whose digit sum is 3 are squares of integers?
10. If the sum of two numbers is 2 and their product is 2, what is the sum of their cubes?
11. GAC. A changing room has 20 lockers numbered 1 to 20. A line of 20 attendeantes P1…P20, file through the room in order. Each attendant Pk changes the condition of those lockers (and only those) whose numbers are divisible by k: if such a locker is unlocked, Pk will lock it and vice versa. Which lockers are unlocked after all 20 attendants pass through the room?
12. What is the geometric mean of 25, 9, and 405? (calculator allowed)
13. Lassiter. Akshaya and Aida had the same quiz grade on the last calculus quiz. For both of the, it was their highest quiz score this semester. It brought Akshaya’s quiz average from an 83 to an 86, and Aida’s from an 88 to a 90 (ownage). How many quizzes has each student in the class taken?
14. Lassiter. Find the sum of all the positive three-digit palindrome numbers.
15. Lassiter. How many perfect numbers between 1 and 100 are also divisible by 3?
16. AMC. The entries in a 3x3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in an increasing order. How many such arrays are there?
17. GAC. Find the sum of *n* terms if the arithmetic series whose first term is the sum of the first *n* natural numbers and whose common difference is *n.*
18. Lassiter. Akshaya needs to pay 2010 dollars to THE SUSHINATOR’S “FUTURE DICTATOR FUND FOR THE UNIVERSE.” She has an unlimited supply of 2, 4, and 10-dollar bills. In how many ways can she pay?
19. N. Fulton. How many more ways are there to place 3 distinguishable balls in 6 distinguishable boxes than there are to place 6 indistinguishable balls in 3 distinguishable boxes?
20. N. Fulton. The US just released the 3 dollar bill to give to poor people. Omolio, who lost all his money from gambling, found an enormous bag of money outside a store containing a total of $1500. After buying food and clothes, he finds a place to live. He sees a nice little apartment for $425 dollars a month. The bag of money contains only 2 and 3 dollar bills, 300 each. How many ways can the rent be paid off?

COOL FACT OF THE WEEK:

There are more Pythagorean triples than you know. Find them. It will help. Greatly.